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A Comparative Analysis of Fractions in Chinese and Pakistani Primary School Mathematics Textbooks

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Abstract

Fractions are a vital content area in school mathematics and are often one of the critical difficulties for schoolchildren to learn during primary school. The study looked at how fractions are treated differently in primary school mathematics textbooks in China and Pakistan, particularly in the early stages of their teaching, which have not been studied. This study proposes a framework for intercultural comparison of textbooks. The results show a higher number of fraction queries in Pakistan than Chinese books, which emphasize the basic definition of fractions to support schoolchildren in developing advanced levels of cognitive abilities. Meanwhile, many graphical representations of fractions are proposed in Chinese to elucidate concepts. Both Pakistani and Chinese textbooks focus on the representation of fractions, together with several symbolic representations of questions. However, the visual content in Chinese math books is relatively high. Besides, the questions raised in Chinese textbooks connected with real-life circumstances, while those in Pakistani textbooks have less relevance to real life. This paper discusses the significance of this research and puts forward some proposals for future research.

Keywords: Fraction Analysis, Math Textbooks, Contextual Analysis.

Introduction

Math textbooks play a vital role in systematizing student knowledge and assisting teachers in teaching math (Cai & Ni, 2011; Fan, Zhu, & Miao, 2013). Textbooks influence not only the content presented and teachers' guidance, but also students' understanding and problem-solving techniques (Fan et al., 2013; Yang & Wu, 2010). Therefore, textbooks play a significant impact on school education and students' knowledge about the course. The researchers used different methods to study how mathematic textbooks around the world create learning prospects for primary school students. Also, scholars have different views on the degree of erudition of mathematics textbooks. Few scholars believe that analysis of textbooks can clarify variances in the educational output of students in an appropriate global educational environment (Son & Senk, 2010; Tan, Ismail, & Abidin, 2018; Yang, 2018). Though, some of them claim that textbooks have a diminutive influence on teaching and students' learning (Stein, Remillard, & Smith, 2007; Tan et al., 2018), and some look textbooks as an imaginable foundation of learning for educators, a goal that is often unattainable (Newton & Newton, 2007; Remillard, 2005). An additional modest view is that schoolbooks provide the

opportunity to learn mathematics with probability rather than certainty (Charalambous, Delaney, Hsu, & Mesa, 2010; Gurganus & Gurganus, 2018; Mesa, 2004). From this point of view, Mesa (2004) calls the textbook analysis a hypothetical initiative: what knowledge would students acquire if their math classes covered all parts of the textbook in a given order? If students had to interpret all the exercises in their math books, what would they learn from it?

The conditional tenses of these binary queries reflect the challenges provoked by textbook critics because the nature of the material in teaching hinges on the way teachers and students cooperation, while this view concedes that textbook analysis solitary studies the proposed syllabus, not the syllabus used, textbooks used by countries may reveal comparisons. Connections and dissimilarities in mathematics education among schoolchildren can reveal the diverse performance prospects of students in several countries, the ranking of the theoretical understanding or practical ease of a country's textbook series, and the ways in which mathematical content is treated differently in each country (Naroth & Luneta, 2015). The fraction is an essential and enlightening topic in the mathematics curriculum of primary schools all over the world. Primary school math textbooks in both China and Pakistan contain fractions. This illustrates the reputation of teaching fractions in primary school math schoolbooks. A fraction is also painstaking to be engrossed by school students, who have several misunderstandings and obstacles in learning, which were discussed in previous literature (Cramer, Post, & delMas, 2002; Ni & Zhou, 2005). In addition, meaningful consideration of the concept of fractions provides a necessary foundation for the study of higher mathematics.

Several studies have shown that schoolchildren's capability to acquire other relevant mathematical impressions is imperfect if they do not have a solid grip on fractions (Charalambous et al., 2010; Yang, 2018; Yang & Wu, 2010). For example, in the United States, fractions are a requirement for algebra achievements, which illustrate the significance of fractions in primary school (Yang, 2018). In addition, Charalambous et al. (2010) noted that "it is proposed that the high performance of East Asian students in international comparative studies is related to differences in textbooks used in these countries". In addition, some studies have only engrossed on subtracting and adding fractions using discriminating sampling, which is not representative (Yang, 2018). Therefore, it can be believed that more research is needed, which prompts us to study the differences in the fraction of mathematics textbooks in primary schools in Pakistan and China.

At present, many studies have equated the differences among Eastern and Western textbooks (Charalambous et al., 2010; Fan et al., 2013; Yang, 2018; Yang, Reys, & Wu, 2010). However, scholars did not conduct a study to explore the differences between Chinese and Pakistani math textbooks. In addition, some investigations have shown that excellent math books can affect students' learning and mathematical procedures (Fan et al., 2013; Yang, 2018). Reys, Reys, & Chavez (2004) point out that the corporatization of mathematics textbooks can influence teachers' teaching methods and, thus, students' learning results. Previous studies have also shown that the style or appearance of mathematical content in textbooks can improve students' learning prospects (Langrall, Mooney, Nisbet, & Jones, 2015; Richland, Stigler, & Holyoak, 2012; Son, 2012; Stein et al., 2007). The way mathematical content is presented in textbooks is important because it is a dynamic "teaching method and different prospects for students to learn" (Stein et al., 2007).

In addition, Zhu & Fan (2006) believes that Chinese students perform well in international mathematics evaluation because they have excellent mathematics courses. However,

Pakistani students have been studying the fraction since the beginning. The research results of this paper have important enlightenment to the outlook and cause of math textbooks in both countries. These conclusions may aid as yardsticks for forthcoming textbook strategies or reconsiderations in both countries and others.

So it would be stimulating to look at the variances between Chinese and Pakistani textbooks. This stirred the authors to perform the study. The motivation of this study was to understand the importance of fractions in math textbooks and to suggest from the research that students could acquire the knowledge about advantages and disadvantages of math textbooks by linking fractions to math textbooks in other countries (Charalambous et al., 2010; Fan et al., 2013; Yang, 2018); Hayek (2018); Aziz, & Mamat (2018). The study aims to equate Pakistan's treatment of primary school fractions with that of mainland China.

Literature Review

Textbook analysis in transnational studies

The study found that there are substantial cross-country differences between math textbooks. The studies were extensively comparing, and contrasting textbooks used in nearly several countries resolved that textbooks fluctuated in several ways and that they showed considerable variances in the presentation and construction of teaching state of affairs (Gilbert, 2003; Mesa, 2004; Remillard, 2005). Small-scale researches have also explored the importance of learning fractions in mathematics textbooks across several countries such as; China, Japan, Finland, Taiwan, and the United States, e.g., (Alajmi, 2012; Fujita & Jones, 2014; Yang, 2018; Yang et al., 2010; Zhu & Fan, 2006) and textbooks used in various European countries (Haggarty & Pepin, 2002; Pepin & Haggarty, 2001).

Inclusively, the study find three categories of cross-country textbook investigation methods, namely "horizontal analysis", "vertical analysis", and "contextual analysis". In the horizontal analysis, textbooks are reviewed as technology throughout the education system (Hiebert et al., 2003), and the general characteristics of textbooks are emphatically analyzed, e.g., appearance, organization of the whole book content. Some criticism of this approach by researchers posits that they ignore the fundamental differences between schoolchildren in different countries in learning opportunities because different textbooks treat and value the subject differently (Yang, 2018). The researcher shows that some ideas were familiarized early in textbooks of one country, while others were not presented in schoolbooks in another country (Cady, Hodges, & Collins, 2015; Charalambous et al., 2010; Son, 2012; Syam, Wijaya, & Retnawati, 2019). While conclusions provide initial visions into the processing of textbook content in those nations, they leave some unanswered queries, among them the main question of how to deal with concepts in the textbook and arrange education prospects for schoolchildren. Therefore, in order to fill this research gap, this study also discusses some other categories and subcategories through the analysis of mathematics textbooks in China and Pakistan.

In contrast, the "vertical analysis" investigates how textbooks consider as a solitary mathematical impression and view textbooks as "the environment for constructing knowledge" (Mesa, 2010). This approach ignores how the action of the topic under review relays on other themes contained in the textbook. So, fraction questions are divided into seven subcategories and compared primary school math textbooks in China and Pakistan. The third method, contextual and non-contextual characteristics analysis, focuses on the way teachers or students use textbooks in teaching activities (Charalambous et al., 2010; Mesa, 2010; Remillard, 2005; Stein et al., 2007) and regarded textbooks as "products in a broad

sense," factually established, culturally shaped, produced for positive purposes, and used with specific intent (Rezat, 2006).

The third method in a straight line helps to solve the fractions in the curriculum implementation, that is, to realize the intention of primary school textbooks, especially in the early stage of curriculum implementation. However, this study have confidence that a framework that comprises both horizontal and vertical analysis is the premise of a contextual analysis of fractions in textbooks (Ding & Li, 2010); Jagero et al., (2016). Coalescing these two analytical aspects reveals the characteristics of the textbook, which are lost by analyzing only one of them.

The lack of common and clear textbook comparison criteria founded in prior studies, making it problematic for scholars to make evocative assessments and to repeat the analysis. Therefore, this study consider it necessary to form a framework that participates in both vertical and horizontal analysis to clarify the content of the analysis and to promote consistency between textbooks and comparisons between countries. Since the progress and implication of this framework is an important section of this relative investigation, which is fleetingly described in detail in the upcoming sections.

Importance of Fractions in Pakistani and Chinese mathematics textbooks

The national mathematics curriculum in both countries is designed differently. Curriculum policies in both China and Pakistan state that students should be given basic learning about fractions at the beginning of their education (Akmal, 2017). A math teacher in Pakistan usually uses textbooks as the main source of arrangement and teaching instruction methods in classrooms. Since the textbooks are designed according to the guiding principle of the national curriculum, this study can simply say that teachers have developed a curriculum for their own classes with the help of the formal curriculum provided by the government (Akmal, 2017; Halai, 2007). However, in the 2009 program for international student assessment (PISA), Chinese students were the best at either overall performance or at a high level of expertise in all contributing nations (OECD, 2010). The growing demand for a balance between globalization and traditions has prompted education representatives to focus on how curricula can attain sustainable progress for young people students (Wang et al., 2012).

Considering the importance of the curriculum, this study incorporated the topic of fractions in this study. The Chinese curriculum defines the basics of a fraction as the foundation to be coached in the third-grade, whereas the Pakistan curriculum defines fractions as fourth-grade. In the third-grade curriculum in China, fractions are taught more broadly, involving concepts such as finding a mutual denominator, executing fractional operations, and converting to decimals and percentages. However, in the fourth-grade curriculum in Pakistan, students are required to sort the fractions in the denominator and resolve queries associated with the addition and subtraction of the fractions in the denominator that will be improved in the upcoming grades (referred from textbooks). Therefore, this study aims to establish a comparative analysis method to reply to the key research question, "How was fraction presented in mathematics textbooks to Chinese and Pakistani students?"

Theoretical Framework

The framework employed in this research was developed in three stages (Charalambous et al., 2010). This study began a comprehensive literature review to recognize harmonies and breaches in the prevailing framework. Succeeding, this study designated the textbook

selection standards applied in prior studies, and finally, divided the standards into categories and subcategories to achieve the current research objectives.

Mainly this study focus on two questions: First, the characteristics of the textbook, such as the number of pages, the theme and the sequence of the theme (that is, using the horizontal method), give the reader a preliminary understanding of whether the textbook is suitable for a certain class of students, or whether the classification, development and comparative focus of the theme meet the requirements of the teacher. However, these features rarely provide detailed information about how the author handles the content. On the other hand, the vertical analysis provides further attention and exhaustive analysis of the mathematical substance of fractions. A framework with two dimensions can benefit from the advantage of each. Second, in each dimension of the analysis, this study can examine different subcategories of fractions (such as the definition of fractions, addition, multiplication, and so on). Therefore, this study tried to establish a framework for analyzing fractions in primary school mathematics textbooks from two dimensions (vertical and horizontal) and focusing on different aspects of the fraction in textbooks. However, any effort to create such a framework is likely to become a compilation of different standards without the fundamental structure or organization (Son, 2012; Yang, 2018). This may bound the framework's ability as a tool for apprehending the understanding of various learning prospects presented to schoolchildren (and teachers) in math textbooks. Therefore, this study divide the standards into categories and subcategories. In the sense of horizontal analysis, this study split the textbook into two categories: "background information and overall structure". The primary category is a vivid outline of the math books and its background — the second category pacts with areas fenced by textbooks and its organizations. This study chose only one textbook from both countries to compare children's learning at the start of fraction education. For the vertical analysis, this study separated the fractions into seven subcategories according to the nature of the questions.

After selecting the framework, this study examined sensitivity to the fractional characteristics of mathematics textbooks in China and Pakistan. This has led to the elaboration of some subcategories and equivalent standards to explain these differences. For example, the example of work in one country is usually fully resolved, while the example of work in another country does not provide the final answer, which challenges the definition of the example of work. In solving these ambiguous questions, this study maintain mathematical integrity by consulting mathematicians and related literature.

Subcategories of the Fraction

Based on the literature review, the seven subcategories related to fractions are demarcated as the "basic definition of the fraction", "ordering of fraction", "equivalent fraction", "addition of fraction", "subtraction of fraction", "multiplication of fraction", and "division of fraction". Relative analysis of fractional addition and subtraction in math books in Cyprus, Ireland and Taiwan (Charalambous et al., 2010) shows that the content of Cyprus and Taiwan "is organized by the nature of Numbers (i.e., similar or different denominators, mixed Numbers) rather than by operation (i.e., addition and subtraction)." Meanwhile, Irish mathematics textbooks report on the operation of fractions with the same and different denominators. They also originate that queries linked to the addition and subtraction of fractions in Taiwanese textbooks had advanced prospects of students than those in math's textbooks in Cyprus and Ireland (Charalambous et al., 2010).

Several fractional-related pieces of research have fixated on the scrutiny of the fundamental knowledge of fractions and the misinterpretations that may happen when students lack sufficient acquaintance of the basic definitions of fractions for more advanced concepts (Braithwaite, Pyke, & Siegler, 2017; Cramer et al., 2002; Kar, Güler, Şen, & Özdemir, 2018). This shows that "the basic definition of fractions" is an important sub-category of fractions in textbooks. Numerous investigations have addressed the subject of the ordering of fractions and discussed student misinterpretations when relating fractions (Braithwaite et al., 2017; Cramer et al., 2002). This means that "ordering fraction" should be considered as an additional important subcategory. Research has examined the key role of knowing the equivalent fractions in ordering fractions, adding, and subtracting fractions (Cramer et al., 2002). Thus, "equivalent fraction" can be thought of as another key subcategory. In addition, some studies examined the performance of students in addition and subtraction of fractions (Charalambous et al., 2010; Cramer et al., 2002; Son, 2012), so another important subclass is "addition" as well as "subtraction" of fraction, and other research have allocated with the thoughts of "multiplication of fraction" and "division of fraction" (Bütüner, 2019; Ding & Li, 2010; Mack, 2000; Son & Senk, 2010), which gives two other key subcategories.

These seven categories are also the main subheadings of the two textbooks. Furthermore, Son 2012) focused on fractions queries associated with addition and subtraction, engrossed on fractions multiplication and division. Charalambous et al. (2010) also focus on adding and subtracting fractions in their studies. Most researches discuss the concept of addition and subtraction in the same category, but they have different properties. Therefore, in this study, this study can observe the addition and subtraction of fractions discretely. Moreover, most of these investigations have engrossed on one particular issue of fractions. Thus, the determination of this research is to reconnoiter all subcategories related to the fraction questions using an analytical framework proposed in this study. Thus, on the base of the said literature review, the fractions questions are divided into seven subcategories.

Representation of Fraction

Grounded on the research of (Zhu & Fan, 2006), and combined with the literature review, this study divides the representation form into "symbol form," "written form", "visual form", and "combined form". Stein et al. (2007) point out that in the two textbook series of the United States, the subject order and presentation are different, which affects students' educational opportunities. Zhu & Fan (2006) believe that offering schoolchildren with diverse types of representation (symbols, writing, and visual), including different types of questions (such as contextual question and non-contextual question), will affect students' learning. They further explored that American math books included a variety of questions, such as visual and contextual questions; Chinese textbooks include more thought-provoking and multi-step queries. Prior research has suggested that diverse forms of expression (symbolic, written, and visual) influence students' learning prospects (Stein et al., 2007; Yang, 2018; Zhu & Fan, 2006). This encourages the authors to study the differences in the forms of expression between the two textbooks.

The Contextual Features of Fraction

Based on the literature review, the contextual characteristics of fractional questions are defined as contextualized and uncontextualized fractional questions. Contextualized and non-contextualized questions are frequently examined in the mathematical textbook analysis, e.g., (Charalambous et al., 2010; Yang et al., 2010; Zhu & Fan, 2006). The diversity and reality

of contextual questions mean that they contain a variety of real-world situations in textbooks. Yang et al. (2010) found that more questions in the mathematical context (MiC) used in the United States than in Taiwan dealing with real-life situations here, also math in Singapore. The authors introduced fractions into the three textbooks, and the content of fractions instructed was nearly a basic level prior to that of these textbooks. This also describes the cultural difference that American mathematical standards are more real than those of east Asian countries. It is fascinating to study the differences between contextual and non-contextual fractions in math textbooks in neighboring countries with different cultures, such as China and Pakistan.

The Features of Textbooks

The uniqueness of the fraction queries in the two textbooks is the characteristic of the textbooks. This uniqueness is when something special appears in one country but not in others. In the study (Yang et al., 2010), they described the symbols used in Singapore math books, but not in Taiwan math books (which only use the words “vertical” and “horizontal”). Because of its uniqueness, this is encoded as a feature of the geometry questions (Yang, 2018). The types of queries stood in textbooks and the manner in which they were posed strongly influenced students’ teaching processes and learning outcomes. Meanwhile, presentation style in textbooks would affect students’ learning opportunities (Cramer et al., 2002; Stein et al., 2007). For example, earlier studies have explored that American textbooks place more emphasis on the use of visual forms to resolve mathematical questions than do Chinese schoolchildren (Zhu & Fan, 2006).

This is because American textbooks contain additional visual questions than Chinese math books. When checking textbook content, examples and exercises are often checked (Charalambous et al., 2010; Fan et al., 2013). Working examples include solutions; the solution presents the problem-solving process, which reports on how the textbook designer presents the solution. Different problem-solving processes also affect students’ learning. Furthermore, some studies have recommended that people can get knowledge about the characteristics of their own mathematic skills by comparing them internationally (Hiebert et al., 2003; Yang et al., 2010). Therefore, this stimulated the researchers to perform this research to distinguish the characteristics of Chinese and Pakistani textbooks.

Methodology

Textbook Selection

Based on the topic of fractions, this study compares mathematics textbooks between China and Pakistan. The subject of fractions begins in Pakistan in grade four and China in grade three. It is important to determine the strength of fraction questions and to compare how fractions are understood in the two countries at the beginning of the course. In addition, dissimilarities were also found in the publication and distribution of math schoolbooks. As there are differences in the schoolbooks between China and Pakistan, it is possible to examine whether there are significant dissimilarities in the characteristics of math books in the two countries. So this study chose books from two countries, and they started teaching fractions. At the beginning of the third-grade, there was only one unit in the Chinese textbooks, and at the beginning of the fourth-grade, there were two units in the mathematics textbooks of Pakistan. To meet the research objectives of the current study, only the Chinese textbooks in grade 3 were compared with the Pakistan textbook in grade 4. In Pakistan, there are 5 math textbooks in total from grade 1 to grade 5 in primary education, and all of the curriculum

content about fractions is in the textbook of grade 4. However, in China, there are 12 textbooks in total from grade 1 to grade 6 in primary education, and fraction starts in the third grade and continues to be taught in the fourth, fifth, and sixth grades. This study have only compared the two textbooks (grade 3 of China, and grade 4 of Pakistan) to compare how fractions are understood in the two countries at the beginning of the course. Because, several studies in the literature have studied the complete course of fraction in primary and elementary education, however, neglected the vital part of discussing the fraction basic-level teaching (Yang, 2018; Zhang, 2016). To find the scope of this study content, this study have only considered exercise questions related to fractions at the beginning that used for students to practice what they have learned.

Analytical Framework

Creating an analytical framework is crucial in carrying out textbook-related research (Fan et al., 2013). In this study, this study presented the examples mostly from the Pakistan textbook because it was written in English; however, some of the examples are also presented from Chinese textbooks. This study applied the following framework based on the exercise:

Counting

Questions associated with fractions in the exercises of student schoolbooks are calculated to extract the total number of fraction stuff. For instance, consider the following: usage of the fraction to represent images, fill in the blanks with appropriate answers, etc.

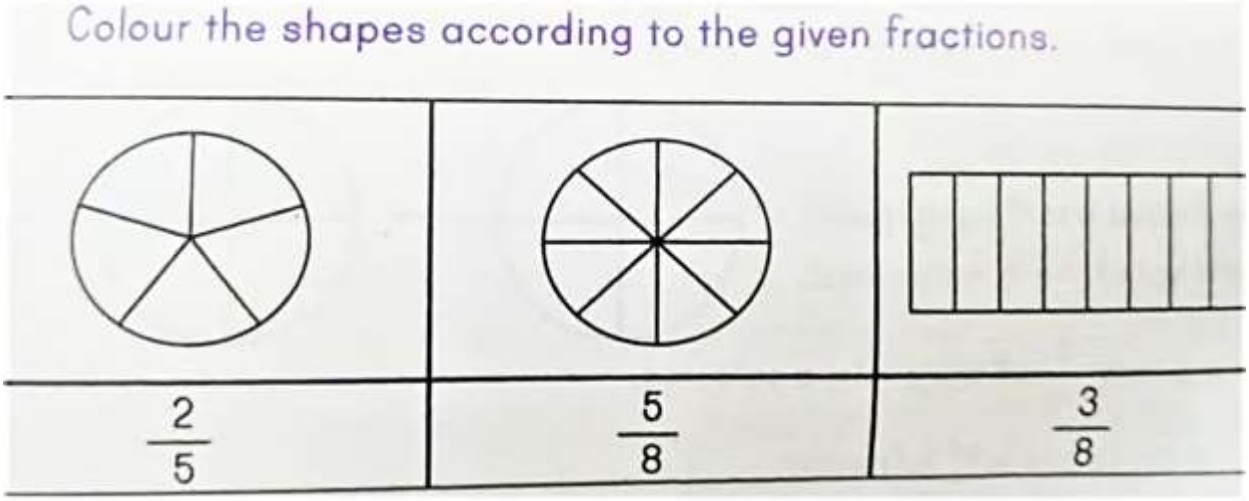


Figure 1 Example for counting *[From grade 4, mathematic textbook of Pakistan P.96]*

For example, in Figure 1, since there are three shapes to fill, this exercise counts as three questions in this analysis of fractions.

Subcategories of the Fraction

Based on the intensive literature reviews and justifications, this study potentially divided the fraction questions into seven subcategories, which are defined below in:

Table 1
Subcategories of fraction questions

Sr. No	Categories of fraction	Details
1	Definition	"This question discusses the basic definition of fractions"
2	Ordering	"This question asks children to compare or sort the size of the fraction"
3	Equivalent	"This question requires students to multiply the numerator and denominator by the same number"
4	Addition	"This question requires students to add fractions"
5	Subtraction	"The question asked the students to subtract fractions"
6	Multiplication	"This question deals with multiplying fractions by numbers"
7	Division	"This question deals with divide fractions by numbers"

Representation of Fraction

The analytical framework of the current paper is also founded on the representative types of fractions in primary school mathematics textbooks, which can be divided into "symbolic form", "written form", "visual form", and "combined form" (Zhu & Fan, 2006). If a fractional question contains only mathematical lingoes, it is encoded in symbolic form, as in

1/2 + 1/2 = ; *[From mathematics textbook of Chinese, p. 97]*

2/9 + 1/3 = ; *[From mathematics textbook of Pakistan, p. 122]*

If a question is asked only in writing, the questions are encoded in writing, for instance:

"一坏巧克力，小东吃了1/8，小红吃了3/8，两人一共吃了几分之几？."

English Translation: A piece of chocolate, Xiao Hong ate 1/8, Xiao Hong ate 3/8, how much two people eat? *[From grade 3. mathematic textbook of Chinese, p. 98]*;

If a question is asked only in figures, pictures, etc., the questions are encoded in visual form, for instance, Figure 2.

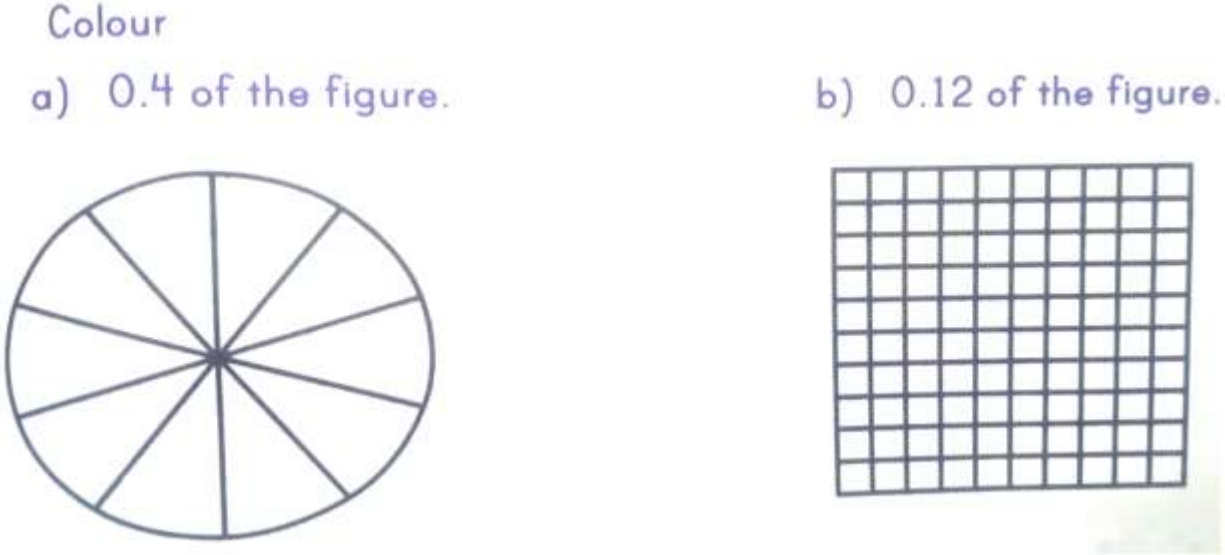


Figure 2 Example of visual form, [*From grade 4, mathematic textbook of Pakistan, p. 133*]

If the books contained two or more of the above forms in the same problem, code it as a combined form. See Figure 3:

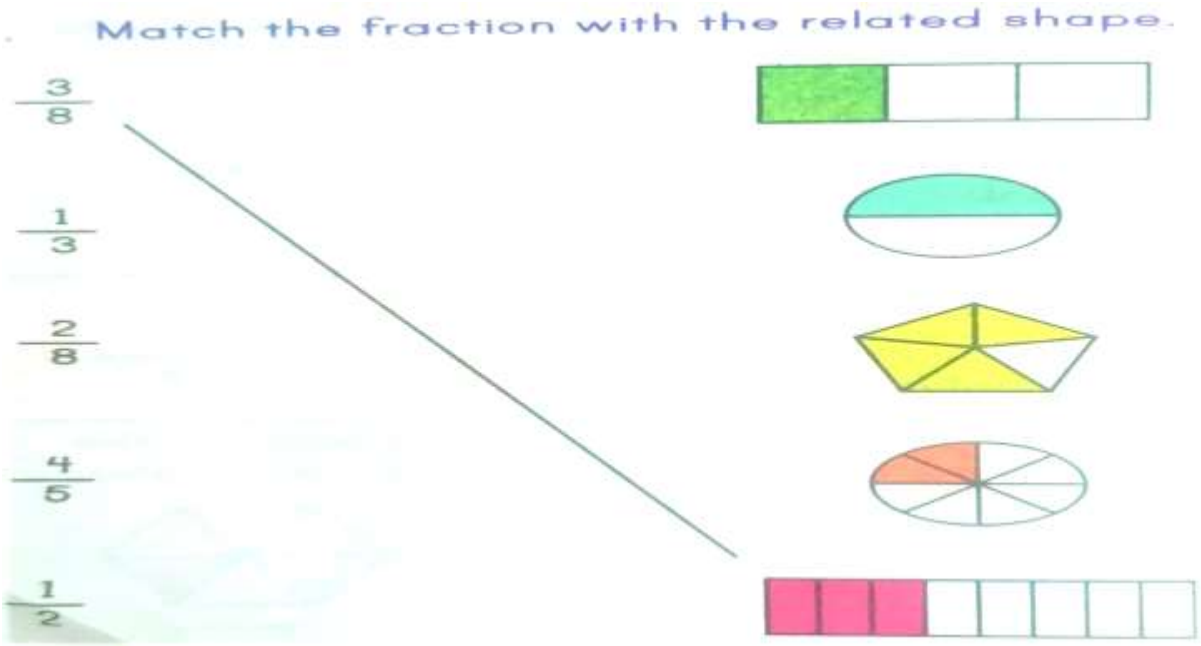


Figure 3 Example of visual forms, *[From grade 4, mathematic textbook of Pakistan, p. 96]*

The Characteristics of the Fraction

The uniqueness of some contents of the question of fractions in the textbooks of the two countries is the characteristic of the fractions questions. This uniqueness is when something special appears in one country but not in others. Because of the uniqueness of the fraction questions, it is encoded as a feature of the fraction questions. Fractional questions are coded as contextual (using real-life context) or non-contextual (using only mathematical language with written symbols) (Yang et al., 2010). This is an example of contextual questions:

"一坏巧克力，小东吃了1/8，小红吃了3/8，两人一共吃了几分之几？"

English Translation: *A piece of chocolate, Xiao Hong ate 1/8, Xiao Hong ate 3/8, how much two people eat?" [From mathematics textbook of Chinese, p. 98]*;

An illustration of non-contextual questions in the textbook is the following;

2/9 + 1/3 = ☐ ; *[From mathematics textbook of Pakistan, p. 122]*

Coding Procedure and Reliability

As mentioned earlier, this study have compared two math textbooks of both selected countries to make a comparison among the practice question at the beginning of fraction education. Therefore, this study only incorporated the exercises where the students asked to solve the fractional problems. As per the coding procedure is concerned, this study have coded the fraction questions based on each fractional exercise. From Figure 4, since there is one exercise unit (unit 2), this exercise counts as six questions in this analysis of fractions. This study considered each fraction as one question from the given exercises. This study set this to define the strength of the fraction questions that students ask at the beginning of their interaction with the fraction, which also helps to understand the teaching level of the fraction and students' understanding. In addition, to verify the reliability of the coding, the two researchers acted as raters to verify the current study coding. Based on the coding framework, the grader coded the contents of the students' textbooks independently, and the final reliability score was above 0.9, which provided a sufficient basis for confirming the reliability of the coding.

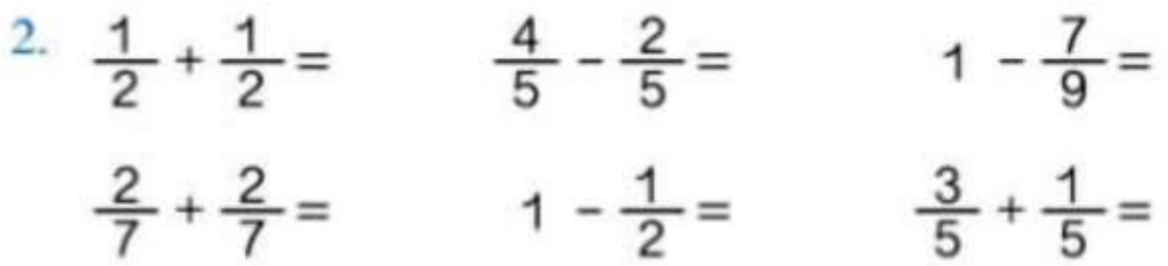


Figure 4 Example of the coding procedure. ***[From the mathematics textbook of China, p. 97]***

Results

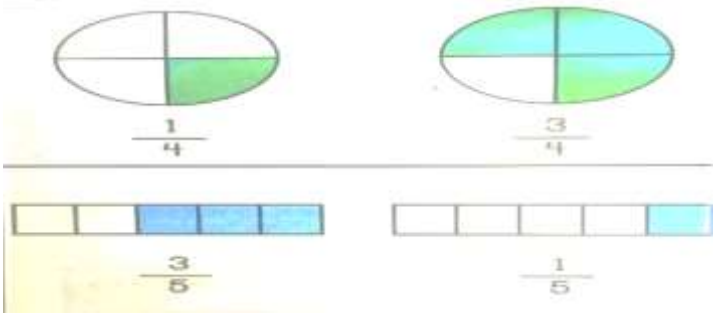
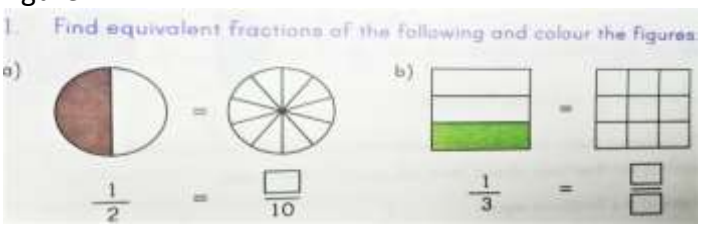
The Differences in the Subcategories of the Fraction

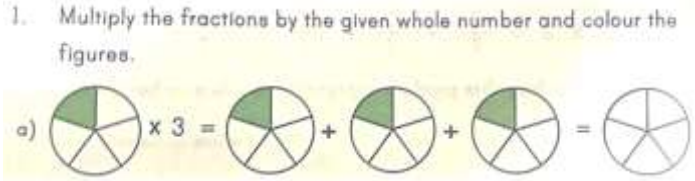
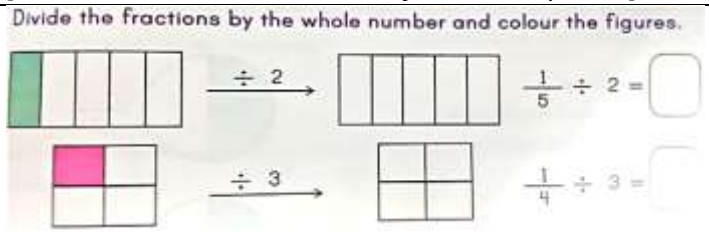
Table 2 lists examples of seven subdivisions of fractional questions in the exercises of two textbooks.

Table 3 shows the percentage and frequency of the difference between the subcategories of fractional questions in the two textbooks. There is a total of 164 questions coded in the Pakistani math book, and 60 questions were obtained from the exercises of the Chinese textbook. According to the data, 63 questions of Pakistan's textbook involved the "basic definition of fractions," which is 38.41% of total questions. While 18 questions of the Chinese textbook involved the basic definition of the fraction, which is 30% of the total questions. Of the seven sub-categories of questions, these are the most common frequency in the two textbooks. Meanwhile, the proportion of fraction "ordering" questions in Pakistani textbooks is very low; about 1.83% (3 questions only), but "ordering" questions are not recorded in Chinese textbooks. In the same category, the proportion of "equivalent of fraction" questions in Pakistani textbooks is higher than that in Chinese textbooks. The total number of "equivalent of fraction" questions in the Pakistani textbook are 37 (22.56%), while in several "equivalent of the fraction" questions in Chinese textbooks are 06 (10%). Compared with the above categories, Chinese textbooks had more questions related to the "addition of

fractions"; having 13 questions containing 21.66% of total questions, while Pakistani textbooks had 22 questions containing 13.41%.

Table 2
Example of seven subcategories of fraction questions

Sr.	Example	Category
1	***Basic definition*** "This question asked about the basic definition based questions"	Which of the following sets show like fractions?  *[From mathematics textbook of Pakistan, p. 98]*
2	***Ordering fraction*** "This is about to compare or order the question"	Arrange the following fractions in ascending order: 3/5 1/5 2/5 = *[From mathematics textbook of Pakistan, p. 101]*
3	***Equivalent fraction*** "This question is about to solve the fraction by multiplying the numerator and denominator by the same number"	Find equivalent fractions of the following and color the figure.  *[From mathematics textbook of Pakistan, p. 115]*
4	***Addition*** "In these questions, students are required to add fractions"	Find the sum of the following, unlike fraction: 2/9 + 1/3 = ☐ *[From mathematics textbook of Pakistan, p. 122]*
5	***Subtraction***	Find the difference of the following unlike fraction: 5/7 - 1/2 = ☐

	"In these questions, students are required to subtract fractions"	*[From mathematics textbook of Pakistan, p. 122]*
6	***Multiplication*** "In these questions, students are required to multiply fractions"	 *[From mathematics textbook of Pakistan, p. 106]*
7	***Division*** "In these questions, students are required to divide fractions"	 *[From mathematics textbook of Pakistan, p. 109]*

As with queries associated with addition, the proportion of "subtraction of fraction" questions in a Chinese math book are 21 (35%), which is also higher proportion than that in Pakistani textbooks containing 25 questions (15.24%). These two series also have a lower proportion of the questions in "multiplication and division of fractions", as shown in

Table 3.

Table 3
Percentage of Questions In Each Subcategory of Fraction Questions

Subcategory	**Pakistan**		**China**	
	Frequency	Percentage (%)	Frequency	Percentage (%)
Definition	63	38.41	18	30
Ordering	03	1.83	0	0
Equivalent	37	22.56	6	10
Addition	22	13.41	13	21.66
Subtraction	25	15.24	21	35
Multiplication	09	5.49	1	1.66
Division	05	3.05	1	1.66
Total	164	100%	60	100%

The proportion of multiplication questions in Pakistani textbooks is 5.49%, while the proportion of multiplication questions in Chinese textbooks is 1.2%. Similarly, Chinese textbooks (1.1%) and Pakistani textbooks (3.05%) have very low grade-related questions compared to other categories. In a word, Pakistani textbooks emphasize the basic concepts of fractions and fraction calculation, while Chinese textbooks focus less on the basic concepts of fractions and more on written form and calculation.

The Differences in the Fraction Representation

Figure 5 demonstrates the representation used in the two textbooks. The expression forms of fractions in primary school mathematics textbooks are symbolic, writing, visual, and combined. In Pakistani textbooks, the symbolic questions are higher than in Chinese

textbooks. Figure 5 shows that over 50% of the questions in Pakistani textbooks are represented by symbols, while symbols characterize over 40% of the questions in Chinese textbooks. Considerable attention has been given to the symbolic representation of issues in Pakistani textbooks.

In addition, the written expression of fractions in Pakistani textbooks is also slightly higher than that in Chinese textbooks, confirming that Pakistani students teach mainly through written statements compared with Chinese students. In comparison with the above statements, the results in Figure 5 show that Chinese textbooks account for a larger proportion in the visual context, which helps to understand the concept of fractions in the initial stage. Visual representation is not at a high level in Pakistani textbooks.

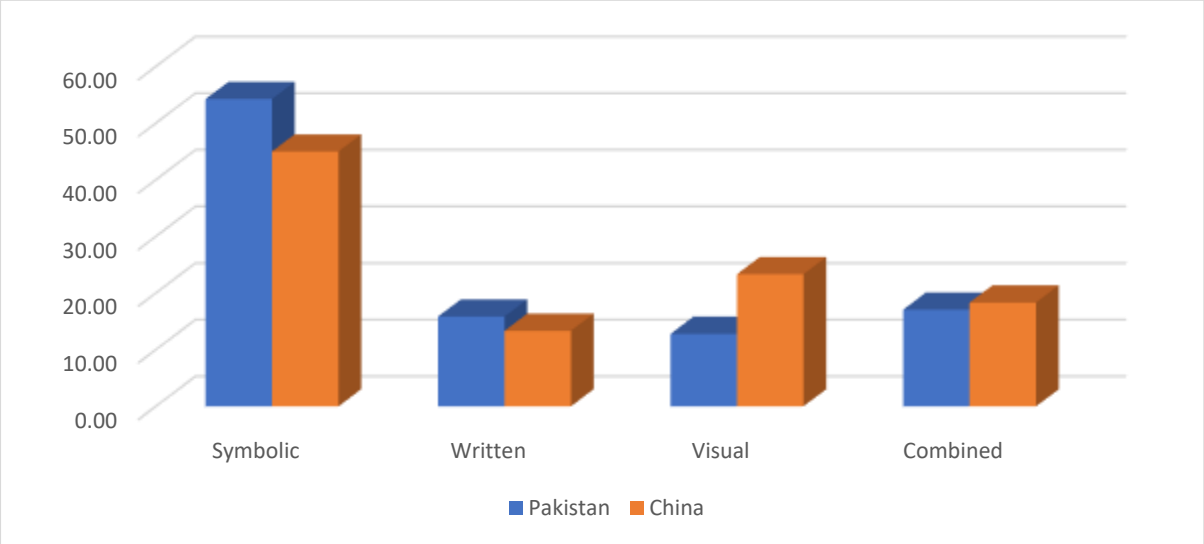


Figure 5 Proportion of representation types two textbooks

The Differences of Context Characteristics of the Fraction

In Figure 6, the percentage of contextual and non-contextual fractions in Pakistani and Chinese textbooks. The two main differences found are described below.

Percentage of Contextual and Non-contextual

The questions in Chinese and Pakistani textbooks are fairly evenly divided into contextual and non-contextual questions in the given exercises. More precisely, on the subject of the basic definition of fractions, in Chinese textbooks, most questions are contextual, and almost 20% are non-contextual, whereas, in Pakistani textbooks, about 80% are non-contextual as shown in Figure 6. This shows that Chinese textbooks pay more attention to contextual questions at the beginning of fraction learning, while Pakistani textbooks only focus on non-contextual questions to define math fractions.

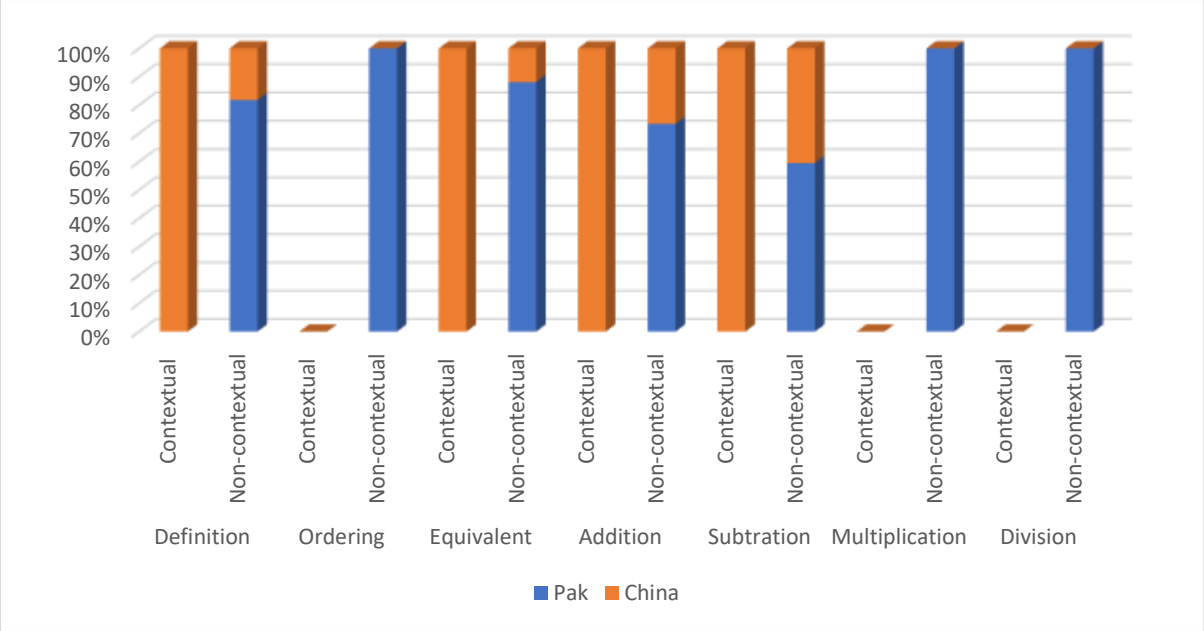


Figure 6 Percentage of contextual and non-contextual questions in two textbooks

In addition, when discussing the second type of fraction questions defined in this study (ordering of fraction), the proportion of ordering questions in both contextual and non-contextual forms in Chinese books of grade three is very low. At the same time, the ordering questions in Pakistani textbooks are mostly non-contextual. Figure 7 also shows that the equivalent categories of fraction questions in Chinese textbooks mainly belong to contextual and 21% approximate to non-contextual. In contrast, Pakistani textbooks only have a non-contextual representation when discussing the equivalent of fractions. The queries in Pakistani textbooks linked to addition and subtraction are all non-contextual representations, while in Chinese textbooks, there are both contextual representations and non-contextual representations.

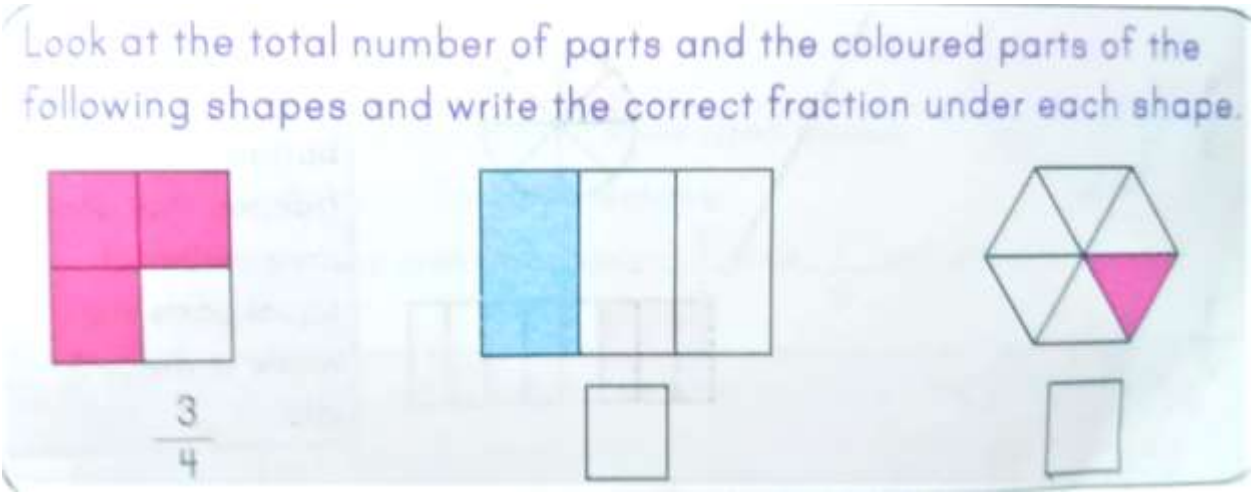


Figure 7 Example of visual questions [*From grade 4, mathematic textbook of Pakistan, p. 95*]

Besides, a similar pattern of contextual and non-contextual proportions in multiplication and division related fractions is shown in the results of this study (see Figure 8).

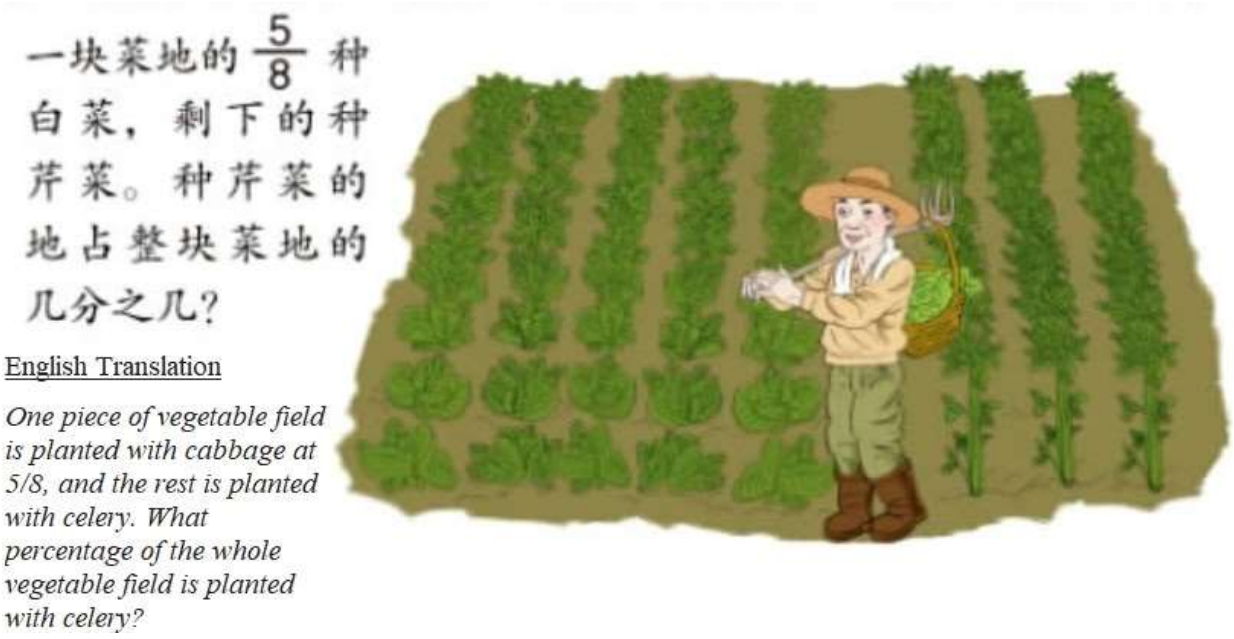


Figure 8 Example of situation questions [*From grade 3, mathematic textbook of China, p. 98*]

Diversity and Realism

Zhu & Fan (2006) divided contextual questions into two categories. "One is the question of human application, where situations and data are set artificially by textbook authors. The other is real application questions, where situations and data are derived from real-life". In Chinese textbooks, vivid illustrations are used to create realistic situational questions for students. For instance, few queries ask schoolchildren to relate their fractions to their routine-life. Others ask schoolchildren to discover, at different intervals, the fraction that represents the activity being performed. An example of this is a question that requires students to use fractions when eating pizza with their parents. Occasionally, students were asked to define the amount of time they spent drawing, the proportion of all the people they worked with, and the proportion of the total amount of food.

In addition, Pakistan's math textbooks have a total of 164 practice questions, compared with 60 in China's. At the beginning of the fractions, Chinese textbooks mainly focus on visual context, but less on practical problem-solving. In addition, the fourth-grade textbook in Pakistan has two units of 37 pages, while the third-grade textbook in China has only one unit with fraction questions. The total length of the fractions unit is recorded in 13 pages in the Chinese textbook. Compared with the very realistic context question expressed by fractions in Chinese textbooks, the contextual questions in Pakistani textbooks lack realism. Moreover, in Pakistani textbooks, simple sentences are used to describe contextual questions, making context less convincing. As a result, it is difficult for students to get the right answer from their own life experience.

Discussion and Conclusion

Discussion

The results showed the difference in the distribution of seven categories of fractions in the Chinese and Pakistani textbooks, where they began to teach fractions. Both textbooks attach great importance to the basic definition of a fraction, but the basic definition of fractions in selected Pakistani textbooks is 8.41 percent higher than that in China. This specifies that the design of textbooks in Pakistan places great importance on developing children's knowledge

of the basic definition of fractions. Similarly, in Chinese textbooks, mostly simple pictures are used when fractions are introduced into learning activities.

The data also show that, initially, textbooks in both countries focused less on "ordering of fractions" questions. Only 1.83 percent of Pakistan's fourth-grade textbooks had ordered fraction questions, while Chinese textbooks did not have any at this stage. There is reason to believe that this may be due to the current emphasis on the written calculation of the basis of fractions (Yang & Li, 2008). Besides, the "equivalent of the fraction" of Pakistani textbooks is 12.56 percent higher than that of Chinese textbooks, which have a 10 percent. This means that both textbooks emphasize the comparison of problems, which may improve children's basic understanding of fractions.

This study also discusses the "addition of fraction" in two textbooks. Contrary to the above results, the Chinese textbook has a higher aggregate of additional questions than the Pakistani textbook. The results showed that 21.67 percent of the questions were related to the addition of fractions in the exercises in Chinese textbooks, while in Pakistani textbooks accounted for 13.41 percent. Similarly, Chinese textbooks have a higher question related to "subtraction of fraction" than Pakistani textbooks. The results showed that Chinese textbooks contained nearly 21 percent more fraction questions than Pakistani textbooks. This shows that the author of the Chinese teaching materials thinks that the development of basic concepts regarding the addition and subtraction of fraction questions is more important than the ordering questions. Also, the results confirm that textbook authors do not pay much attention to multiplication and division of fractions in the early stages of math learning in both countries. As a result, both textbooks have a low proportion of fractions multiplication and division, but Pakistan has a higher proportion of such questions than Chinese textbooks.

Based on the expressions of fractional problems in mathematics textbooks of China and Pakistan, this study divides fractional questions into four categories: symbolic, written, visual, and combined representation of fraction questions. The results show that both countries are concerned about symbolic issues. According to the results from Figure 5, the proportion of symbolic questions in textbooks of both countries is higher than that of other representation. However, the proportion of textbooks in Pakistan is slightly higher than in China. These two textbooks attach great importance to the mastery of practical acquaintance of fractions through such representation, because, at an early stage, it is also a challenging problem in developing a common and strong understanding of fractions among children. The results show that compared with other representation methods, the two textbooks pay less attention to the written representation of a fraction.

The results show that the visual forms in Chinese textbook fraction questions are diverse, interesting, and helpful to the development of understandings associated with fractions. In contrast, the use of visual forms in Pakistani textbooks is less sticky, even restricted to a few basic shapes. This way of thinking will make it difficult for students to understand the concept of fractions. Cramer et al. (2002) argue that the conceptualization of fractions can be established over the connection between "visual representations" and "symbolic representations". In addition, Yang & Wu (2010) found that "visual representation" can be a useful tool to help children develop the concept of fractions. Moreover, the research of Wang & Siegler (2013) originates that visual representation can expand the psychological representation of fractions in students. In light of the above outcomes, it can be determined that the "visual forms" used in Pakistani math books could be upgraded.

This study also employed a "combined" method to distinguish the performance of fractional problems, which may be measured, while two of the three methods above are used for the

same fraction questions. This will add to the basic understanding of the fractional questions. The results show that the emphasis of the two textbooks is virtually the same but also having some dissimilarity at various points.

Conclusion

The study looked at the different ways fractions are treated in primary school mathematics textbooks in China and Pakistan. Although the number of two textbooks in both countries is limited, the results describe a few imperative and exciting variances. In this study, this study found several foremost differences between the textbooks of Chinese and Pakistani curriculum.

First, Chinese textbooks not only highlight the basic concept of fractions but also help children develop higher levels of reasoning. Secondly, the fraction questions in Chinese textbooks are expressed more clearly by a wide range of visual and contextual representations. Thirdly, the questions used in Chinese textbooks are closely related to routine life, which in turn helps children perceive the relevance of mathematics when they are first exposed to it. Finally, Pakistani textbooks use a range of different symbolic representations, while Chinese textbooks focus on symbols and visual presentations. These variances should be taken into account in the upcoming creation and development of curricula in those countries and could be implemented in other countries.

Contributions, limitations, and Future Research

Contributions

Theoretical contribution

The study findings also contribute to the existing curriculum literature. The differences found in this study in two textbooks help to understand these questions in the literature: how is the topic of fractions determined in textbooks of different countries? What type of questions is important to develop an understanding of fractions in primary schools? At the same time, the framework proposed in this paper is also likely to be replaced and studied, which will also contribute to the cross-cultural analysis of textbooks in the literature.

Practical Contribution

In this context, the practical contributions of these findings are also observed as; the realistic math questions in Chinese textbooks can help children realize the significance of math in routine life. Since the problems raised in these schoolbooks are severely associated with real-life circumstances, students can easily comprehend that a small part of what they learn in math class can be employed to resolve real-life math problems. In this setting, when schoolchildren are first exposed to fractions, they can deeply feel the practicality of mathematics in their lives. This, in turn, can boost schoolchildren's enthusiasm and concentration in math, especially when it comes to questions related to fractions. By contrast, the queries postured in Pakistani math schoolbooks do not demonstrate accurate circumstances but emphasize repetition to create schoolchildren's numeracy skills. In such a knowledge atmosphere, it is difficult for students to realize the significance of fraction applications in real-life, thus increasing their interest and motivation in the subject.

Limitations

Besides the significant contribution of this study, there are few limitations. First of all, this study only focused on the comparison of fraction topics only in China and Pakistan have two

different linguistic and cultural values. Generalizations about the conclusions had better be treated with carefulness, as the research only looked at two dissimilar textbooks in the two countries. Secondly, the results of current research demonstrate that there are variances in the design of seven subdivisions of fractions, different types of representations, and contextual differences. Though schoolbooks are seen as key materials for teaching math in the classroom, teachers can play a significant role in developing the learning attitudes in the classroom. Therefore, generalizations about these differences should be made with caution, as experienced teachers may change these differences due to some cultural values and teaching habits.

Future Research

Further study is required to address issues such as in what way teachers in China and Pakistan use math textbooks in the classroom to establish a basic concept of fractions. Also, because of these problems, the relationship between mathematics textbooks, students' mathematics outcomes, and teachers' instruction would be studied and examined longitudinally among several neighbor countries. In addition, these findings have implications for the fullness of the arena of cross-country textbook analysis, and it's possible to subsidize to developing a common consideration of teaching and student learning perspectives.

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